A Note on Additions to Menchero's Multiperiod Arithmetic Attribution Methodology

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1 Stock-Level Performance Attribution

1.1 Introduction

This document details the stock-level attribution methodology implemented in our Python package for performance attribution. While the package primarily implements Menchero's multiperiod arithmetic attribution methodology (Menchero 2004), it extends beyond the original paper in two key aspects:

1. It provides stock-level attribution effects that properly aggregate to sector-level effects
2. It incorporates trading profits into the attribution framework

Menchero's paper focuses on sector-level attribution and linking of effects through time. This note explains our specific approach to handling stock-level attributions and trading profits within the broader Menchero framework.

1.1.1 Treatment of Interaction Effect

While Menchero's methodology presents a three-factor decomposition (allocation + selection + interaction), this implementation combines the interaction effect with the selection effect, resulting in a two-factor model. This simplification maintains the mathematical integrity of the attribution while streamlining the interpretation of results.

1.2 Initial Calculations

At the stock level, initial attribution effects are calculated using the standard Brinson formulas, analogous to sector-level calculations. For a stock i in sector j at time t:

$$\text{Selection Effect} = w_{i,t}(r_{i,t} - \bar{r}_{i,t}) \tag{1}$$

$$\text{Allocation effect} = (w_{i,t} - \bar{w}_{i,t})(\bar{r}_{i,t} - \bar{R}_t) \tag{2}$$

where:

- $w_{i,t}$ is the portfolio weight of stock i
- $\bar{w}_{i,t}$ is the benchmark weight of stock i
- $r_{i,t}$ is the portfolio return of stock i
- $\bar{r}_{i,t}$ is the benchmark return of stock i
- $\bar{R}_t$ is the total benchmark return

1.3 Effect Redistribution

After initial calculation, the effects must be modified to ensure proper aggregation to sector-level attributions. This modification involves two key steps:

1.3.1 1. Allocation Effect Distribution

Sector-level allocation effects are distributed to constituent stocks proportionally based on normalized portfolio weights:

$$Allocation_{i,t} = Allocation_{j,t} \times \frac{w_{i,t}}{\sum_{k \in j} w_{k,t}} \tag{3}$$

where:

- $Allocation_{i,t}$ is the final stock-level allocation effect
- $Allocation_{j,t}$ is the sector-level allocation effect
- $\sum_{k \in j} w_{k,t}$ is the sum of portfolio weights in sector j

This ensures that stocks with larger portfolio weights within their sector receive a proportionally larger share of the sector's allocation effect.

1.3.2 2. Effect Swapping

After distributing the sector allocation effects, the stock-level effects are swapped: - Initial stock-level selection effects become allocation effects - Initial stock-level allocation effects become selection effects

This swap is necessary to maintain mathematical consistency across hierarchies. The final attribution effects satisfy:

$$\sum_{i \in j} Allocation_{i,t} = Allocation_{j,t} \tag{4}$$

That is, stock-level allocation effects within a sector sum to the sector's allocation effect.

1.4 Trading Profit Treatment

The package incorporates trading profits into the attribution framework, another extension to Menchero's methodology. Trading profits ($PnL_{i,t}$) are included in the portfolio return calculations:

$$r_{i,t}^{total} = r_{i,t} + PnL_{i,t} \tag{5}$$

where:

- $r_{i,t}^{total}$ is the total return including trading profits
- $r_{i,t}$ is the standard portfolio return
- $PnL_{i,t}$ is the profit/loss from trading activity as a percentage of portfolio value

Trading profits are attributed entirely to the selection effect, reflecting that they result from active trading decisions rather than static allocation choices. This treatment ensures:

1. Full accountability of all sources of return
2. Proper attribution of trading activity to active management decisions
3. Maintenance of the mathematical relationship between stock and sector-level effects

1.5 Rationale

The redistribution and swapping of effects serves several purposes:

1. It ensures mathematical consistency between stock and sector-level attributions
2. It properly attributes sector allocation decisions to the constituent stocks
3. It maintains the interpretability of attribution effects across hierarchical levels

This approach recognizes that while stock-level calculations can use the same mathematical formulas as sector-level calculations, the interpretation and aggregation of these effects requires careful consideration of the hierarchical relationship between stocks and sectors.

References

Menchero, J. 2004. "Multiperiod Arithmetic Attribution." *Financial Analysts Journal* 60 (4): 76–91.